Course Code: PGMAT4C006T

Course Title: Operator Theory:

Objectives: The course's main goal is to study the fundamentals of operator theory. It is a field that has great importance for other areas of mathematics and physics, such as algebraic topology, differential geometry, and quantum mechanics.The classical areas of operator theory are the spectral theory of linear operators, distribution theory, operator algebra theory,the geometry of Banach spaces etc. It has numerous applications in differential equations, harmonic analysis, representation theory, geometry, topology, calculus of variations, optimization, quantum physics, etc. It assumes a basic knowledge in functional analysis but no prior acquaintance with operator theory is required.

Learning Outcomes :

The students will be introduced to topics of operator theory with an emphasis on spectral theory and to the fundamentals of Banach algebra theory.In particular, students should be able to do the following after completing this course:

- find the strong, uniform and weak convergences;
- prove the continuity of concrete linear operators between topological vector spaces;
- prove whether a linear operator is compact or not;
- find the essential spectra of linear operators;
- find the maximal spectra of concrete commutative Banach algebras;
- describe the functional calculi and the spectral decompositions of concrete self-adjoint operators;
- find the bounded operators on Hilbert spaces.

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Course Title: Operator Theory **Duration of Examination: 3 hours**
Course Code: **Maximum Marks: 100**
Course Credits: 4

Unit-1

• Spectral Theory in Finite normed spaces: Definition of eigenvalues, eigenvectors, eigenspaces, spectrum, resolvent set of a matrix, eigen values of an operator, Existence Theorem for eigenvalues, spectral theory for infinite dimensional normed linear spaces, resolvent of an operator, spectrum of a bounded linear operator on a complex Banach space, Representation Theorem, Resolvent equation, commutative properties of resolvent, Spectral Mapping Theorem for Polynomials, definition of local holomorphy, holomorphy of resolvent operator, Spectral radius of a bounded linear operator.

Unit-2

• Definition of normed algebra, Banach Algebra and examples, invertible elements, Banach-Alaoglu theorem (statement only), multiplicative linear functional, definition of spectrum, resolvent set, spectral radius, division algebra, Gelfand Mazur Theorem, Spectral Mapping Theorem.

Unit-3

• Definition of compact linear operator on normed spaces, examples, compactness criterion, uniform limit of a sequence of compact operators, finite rank operator, eigenvalues and eigenspaces for compact operators.

Unit-4

• Unbounded Linear Operators: Hellinger-Toeplitz Theorem, Densely defined operators, Hilbert-Adjoint operators, Inverse of the Hilbert-adjoint operator, Symmetric linear operator, closed linear operator, definition of closable operator, closure, spectrum of self-adjoint linear operator.

Unit-5

• Multiplication operator and Differentiation operator, self-adjoint multiplication operator, spectrum of multiplication operator, definitions of States, Observables, Position operator and Moment operator, Heisenberg Uncertainty Principle.

Textbook:

1. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, USA, 1989.

Reference books:

1. Ronald G. Douglas, Banach Algebra Techniques in Operator Theory, Springer-Verlag, New York, 1998.
2. John B. Conway, A course in Functional Analysis, Second Edition, Springer, 1990.
3. Arch W. Naylor and George R. Sell, Linear Operator Theory in Engineering and Sciences, Springer-Verlag, New York, 2000.

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Course Code: PGMAT4F006T

Course Title: Stochastic Processes

OBJECTIVES: The goal of the course is to give students a fundamental understanding of stochastic processes, particularly Markov processes, as well as a foundation for using stochastic processes as models in a wide range of applications, including queueing theory, Markov chain Monte Carlo, and their applications in modern engineering problems.

Learning Outcomes

Upon successful completion of this course, students will be:

- Describe stochastic processes in detail, particularly Markov processes, after successfully completing this course.
- Able to define Markov chains in both discrete and continuous time.
- Able to establish a good understanding of discrete state Markov processes such as Markov chains, Poisson processes, and birth and death processes, as well as queueing systems.
- Able to formulate simple time-domain stochastic process models and perform qualitative and quantitative assessments on them.

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Name of the Course: Stochastic Processes **Course Credits: 4**

Course No.:

Unit-I

Stochastic or random processes definitions, applications and examples, continuous and discrete stochastic processes,, Stationary and Evolutionary stochastic processes, Distribution and density functions, mean, correlation, covariance and auto-covariance functions, Probability generating function (pgf) method, some elementary exercises.

Unit-II

Markov processes and Markov chain, Homogeneous Markov chain, Transition probability matrix, one step and n-step transition probability, Steady state condition on Markov chain, Classification of Markov chain, Some Markov chain models, Chapman-Kolmogorov theorem, Regular Markov chain, Irreducible Markov chain, Periodicity, Ergodicity.

Unit-III

Poisson Processes, Poisson distribution, Distribution associated with the Poisson Processes, Properties of Poisson processes, The Law of Rare Events and the Poisson Process, Pure Birth Processes, Pure Death Processes, Distribution of inter arrivals and departures, Yule-Furry process, Simple Birth and Death Processes.

Unit-IV

Queueing Processes, Basic Queueing characteristics, Kendall's Notation, Steady state distribution, Poisson arrivals, Exponential service times, Little's formula, M/M/1: ∞ queueing model, M/M/c: ∞ queueing model, M/M/1: K queueing model and their performance measures, M/M/c: K queueing model, Examples based on these models.

Unit-V

M/M/1: ∞ queueing model with state dependent service rates, Finite population M/M/c/K/K queueing model, Transient behavior of M/M/1/1 model, Non-Markovian queueing models as M/G/1 and various performance measures, Queue Networks, Series queues with blocking and their performance measures,

Reference Books:

1. **Howard M. Taylor and Samuel Karlin**, "An Introduction to Stochastic Modeling", Third Edition, Academic Press, London.

2. **B. R. Bhat,** "Stochastic Models: Analysis and Applications", First Edition, (Reprinted on Jan, 2000), ISBN No.: 978-81-224-1228-4, New Age International, Delhi.

3. **J. Medhi,** "Stochastic Processes", Third Edition, New Age International, Delhi.

4. **S. Palaniammal,** "Probability and Queueing Theory", PHI Learning Private Limited, Delhi.

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Course Code: PGMAT4C005T
Course Title: INTRODUCTION TO CRYPTOGRAPHY

OBJECTIVES:

To make the student learn different encryption techniques along with hash functions, MAC, digital signatures and their use in various protocols for network security and system security.

LEARNING OUTCOMES

- Analyze and Design Classical encryption techniques and block Ciphers.
- Understand and analyze data encyption standards.
- Understand and analyze public-key Cryptography, RSA and other public-key cryptosystems.
- Understanding Protocols
- Analyze and design digital signatures.
- Design network application security schemes.

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Semester-4

Course Title: Introduction to Cryptography **Course Code: ....**

Duration of Examination: 3 hours **Credits: 04** **Maximum Marks: 100**

Prerequisite: Some basic knowledge of Number Theory, Set Theory and Lattice Theory will help to understand the course.

Unit-1

Introduction to cryptography: Private and Public key Cryptosystems, Classical Cryptography, Simple Substitution Ciphers, Cryptanalysis of Simple Substitution Ciphers, Cryptography before the Computer Age, Symmetric and Asymmetric Ciphers, An Encoding Scheme and an Encryption Scheme, Symmetric Encryption of Encoded blocks, Examples of Symmetric Ciphers, Random bit sequence and symmetric ciphers.

Unit-2

Introduction to asymmetric ciphers, Origin of Public Key Cryptography, The Discrete Logarithm Problem, Diffie-Hellman Key Exchange Algorithm, The ElGamal Public Key Cryptosystem, Hardness of the Discrete Logarithm Problem, Order Notation, A Collision Algorithm for Discrete Logarithm Problem.

Unit-3

Integer factorization and the RSA cryptosystem: Euler's formula, Roots modulo pq with p & q as distinct primes, The RSA public key cryptosystem, It's implementation and security issues, Primality testing, Miller-Rabin test for composite numbers, The prime number theorem (statement only), Riemann zeta function, Riemann hypothesis, AKS primality test (statement only), Pollard's $p-1$ factorization algorithm.

Unit-4

Legendre's symbol, quadratic reciprocity, Jacobi symbol, Probabilistic encryption and the Goldwasser-Micali cryptosystem. Information theory: Perfect secrecy, Conditions for perfect secrecy, Entropy, Redundancy and the entropy of natural language, The algebra of secrecy systems, Complexity theory and P versus NP.

Unit-5

Digital signatures: Definition, Components of a digital signature scheme, RSA digital signatures, ElGamal digital signatures digital signature algorithm (DSA), GGH lattice based digital signature scheme, NTRU digital signature scheme.

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Textbook: J Hoffstein, J Pipher & J H Silverman, An introduction to mathematical cryptography, Springer (India) Pvt. Ltd., 2011

Reference Books:

1. V V Yaschenko, Cryptography: An introduction, American Mathematical Society, 2009
2. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 2008, 6th Ed.,
3. J Talbot and D Welsh, Complexity and Cryptography: An Introduction, Cambridge University Press, 2006

Kamlesh Kumar

Course code:

Course title: Discrete Mathematics

Course Credits: 4

Unit-1

- Set, Relation, Well-ordering principle, Poset, Lattices, trees, Boolean Algebra, Boolean functions, disjunctive and conjunctive normal form, Representation theorem for Boolean Algebra, Representation of relations as digraphs and Boolean matrices.

Unit-2

- Combinations, Counting Principles, Binomial Coefficients, Set-partitons and Stirling number of first and second kinds, Number partitions, Ferrers diagram

Unit-3

- Inclusion-Exclusion Principle and its Applications
- Generating function, Solving Recurrence relations, Convolution, Exponential generating function, Dirichlet generating function.

Unit-4

- Graph Theory: Graphs, subgraphs, Isomorphism of graphs, Adjacency and Incidence matrices; Trees, Forests, Counting labeled trees, Spanning subgraphs, Kruskal's algorithm
- Matching theory, Hall's Marriage Theorem

Unit-5

- Planar graphs, Euler's formula, Five colour theorem, Chromatic polynomial of a graph, Edge colourings, Hamiltonian Cycles, Ramsey Theory, Diameter and eigenvalues of a graphs.

Recommended Texts.

1. Richard A. Brualdi, Introductory Combinatorics, 5th edition, Pearson Education.
2. Ralph P. Grimaldi, Discrete and Combinatorial Mathematics: An Applied Introduction, 5th edition, Pearson Education, 2003.
3. Sebastian M. Cioaba and M. Ram Murty, A first Course in Graph Theory and Combinatorics(Texts and Readings in Mathematics), Hindustan Book Agency, 2009.
4. C L Liu and D P Mohapatra, Elements of Discrete Mathematics, McGraw Hill, 1985.

P. Singh Kamlesh Kumar

TOPOLOGICAL VECTOR SPACE

UNIT-1

Semi norm, Topological vector space, Convex set, Balanced set, Absorbing set, Minkowski functional (Gauge),Topology in a semi- norm, Linear space, Semi normed linear space, Locally convex space.

UNIT-2

Linear Transformation, Linear functional, Maximal subspace, Linear variety, Hyper plane, Geometric form of Hahn Banach theorem.

UNIT-3

Reflexive Banach space, Cannonical embedding, Milman's theorem, Weak Topology, Basic neighbourhoods, Weak*-topology, F-topology, The Banach Alaoglu theorem, Extreme Points, Extremal Subset.

UNIT-4

Krein-Milman theorem, Baire's Category theorem, Closed Graph theorem, Application of Closed Graph theorem, Frechet space, Open Mapping theorem for Frechet space.

UNIT-5

Absolutely convex set, Duality, Linear form, Weak topology, Polar of a set, Bipolar theorem, Barrelled Space, Bornivorous or Bornivorne, Bornological Space.

Books:

1. **Larsen, R.,** *Functional Analysis: an introduction*, M. Dekker 1973.
2. **Schaefer, H.H.** ,*Topological Vector Spaces*, Springer 1999.
3. **Rudin, walter,**Functional Analysis (2nd edition),McGraw-Hill,1991

Kamlesh Kumar

Course Title: Fourier Analysis **Course Code:** C.

Duration of Examination: 3 hours **Maximum-Marks: 100**

Unit-1

- Lp- spaces. Holder's inequality and Minkowki's inequality (statements only), completeness OF Lp –spaces, Approximations by continuous functions. Fourier series, Fourier sine series and Fourier cosine series, Smoothness, the Riemann-Lebesgue Lemma, the Dirichelt and the Fourier Kernels, Area under Dirichlet Kernel on $[0, \pi]$, the Riemann- Labesgue property of the Dirichelet Kernal , Continuous and Discrete Fourier Kernal.

Unit-2

- Pointwise convergence of Fourier Series, criterion for pointwise convergence, Dini's test Lipschitz's test, Selector property of $[\sin(n + \frac{1}{2}u/u)]$, Dirichlet point-wise convergence theorem, the Gregory series, Selectór property of $(\sin w)/t$, point-wise convergence for B V, uniformly convergent trigonometric series and Fourier series, Absolutely convergent coefficients, Uniform convergence for piecewise smooth functions.

Unit-3

- The Gibb's phenomenon, the Gibb's phenomenon for a step function, Divergent Fourier series, Term-wise integration and term-wise differentiation, Trigonometric vs. Fourier series , Smoothness and speed of convergence, Dido's Lemma, other kinds of summability, Toeplitz summability, Toeplitz summability , abel summability.

Unit-4

- Fejer Kernal, Properties of Fejer Kernal, Fejer's Theorem, Lebesge pointwise Convergence Theorem. The finite Fourier Transform, convolution on the circle groupT.

Unit-5

- The exponential form of Lebesgue theorem, Labesgue's pointwise convergence Therom -II, the Fouier transform and residue , the Fourier map, Convolution on R, Inversion, Exponential form and Trigonometric form.

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Textbooks:

1. George Bachman, Lawrence Narici and Edward Beckenstein Fourier and Wavelet Analysis, Springer – Verlag, New York, 2005

Reference books:

1. C S Rees, S M Shah, C V Stanojevic: Theory and applications of Fourier Analysis, Marcel Dekkar Inc., New York
2. Rajendra Bhatia, Fourier Series, Hindustan Book Agency, Delhi.
3. N. K. Bary, A Treatise on Trigonometric Series, Pergamon Press., A. Zygmund, Trigonometric Series, Cambridge Press.
4. H P Hsu, H B Jovanovich, Applied Fourier Analysis, New York.
5. K G Beauchhamp, Walsh Functions and their applications, Academic Press.
6. E O Brigham, The Fast Fourier Transform, Prentice Hall of India.

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