Course code:

Course title: Fields and Galois Theory

Course Credits: 4

Unit 1

- Field: Definition and Examples, Types of fields, algebraic and transcendental elements, minimal polynomial of an algebraic element, simple extension
- Degree of a field extension, multiplicative property of degree, Classification of Quadratic extensions,

Unit 2

- Straight edge and compass constructions: constructible numbers, degree of a constructible number, trisection of an angle, Construction of regular polygon
- Splitting Field of a Polynomial, Existence and Uniqueness, Multiple Roots, Perfect Field

Unit 3

- Galois group, Examples, Artin Lemma, Separable and Normal extensions, Galois extension, Fundamental Theorem of Galois Theory
- Fundamental Theorem on symmetric polynomials, Symmetric rational functions

Unit 4

- Solvability by radicals, Galois group of a polynomial, Cyclotomic field and its Galois group
- Galois group as permutation group of the roots

Unit 5

- The general equation of the n^{th} degree, Abel-Ruffini Theorem, Equations with rational co-efficients and Symmetric group as Galois group
- Constructible regular n-gons, Cyclotomic fields over $\mathbb{Q}$.

Recommended Texts.

1. N Jacobson, Basic Algebra, Vol.- I, second edition, Dover Publications, 2012
2. P B Bhattacharya, S K Jain, S R Nagpaul, Basic Abstract Algebra, Second edition, Cambridge University Press, 1994.
3. M Artin, Algebra, Second edition, PHI Learning Private Limited, New Delhi, 2012.
4. I N Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

P. Singh Kamlesh Kumar

Alka

Course Code: PGMAT3C005T

Course Title: FUNCTIONAL ANALYSIS.

OBJECTIVE : The objective of this course is to introduce basic concepts, methods of Functional Analysis and its Applications. It is a first level course in Functional Analysis.It is a core course in any mathematics curriculum at the masters level. It has wide ranging applications in several areas of mathematics, especially in the modern approach to the study of partial differential equations.The proposed course will cover all the material usually dealt with in any basic course of Functional Analysis. Starting from normed linear spaces, it covers all the important theorems, with applications, in the theory of Banach and Hilbert spaces.

LEARNING OUTCOMES: After the completion of course the students will acquire knowledge on the following:

- They study vector spaces which are endowed with a topology, in particular infinite-dimensional spaces.
- They are able to know the extension of the theory of measure, integration, and probability to infinite dimensional spaces, also known as infinite dimensional analysis.
- They will be able to compare the differences between finite and infinite dimensional spaces.
- Also, they will know how to compare the differences between Banach and Hilbert spaces.
- Analyse the structure of the spectrum of certain operators.
- They will know how touse topology to work with infinite dimensional vector spaces.

Kamlesh Kumar

Course Title: Functional Analysis **Duration of Examination: 3 hours**
Course Code: **Maximum Marks: 100**

Objective: Functional Analysis plays an increasing role in the Applied Sciences as well as in Mathematics itself. Consequently, it becomes more and more desirable to introduce the student to the field at an early stage of study.

Unit-I

Normed linear spaces, Banach Spaces and examples, subspace of Banach space, Completion theorem, properties of finite dimensional normed linear spaces and subspaces, equivalent norms, compactness, F. Riesz's Lemma, linear operators and examples, Inverse operator.

Unit-II

Bounded and continuous linear operators and examples, properties of bounded linear operators, relations between bounded and continuous linear operators, linear functionals and their properties, dual spaces. Find duals of R, c, l , 0 < p < infinity.

Unit-III

Hahn-Banach Theorem for real linear spaces, Complex linear spaces and normed linear spaces, Adjoint operator, Reflexive spaces, Uniform Boundedness Theorem, Baire's Category Theorem(statement), Strong and weak convergence, convergence of sequences of operators and functionals, Open mapping Theorem, Closed Graph Theorem with examples and counter examples.

Unit-IV

Inner product spaces, Hilbert space, parallelogram Law, Orthogonal Complements and Direct Sums, orthonormal sets and Sequences, Pythagorean Relation, Bessel's inequality, series related to orthonormal sequences and sets, Total orthonormal sets and sequences, Separable Hilbert spaces.

Unit-V

Legendre, Hermite and Laguerre Polynomials, Riesz's Representation Theorem, Hilbert adjoint operator, properties of Hilbert adjoint operator, Reflexive spaces, Self-adjoint, unitary and normal operators.

Text book:

- Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, USA, 1989.

Reference books:

- George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill International editions.
- Martin Schechter, Principles of Functional Analysis, AMS, second edition, 2002.
- John B. Conway, A course in Operator Theory, AMS, 2000.
- Balmohan V. Limaye, Functional Analysis, New age International (P) Limited, Publishers, second edition, 1996.

Kamlesh Kumar

Course Code: PGMAT3E006T

Course Title: FINITE FIELDS AND CODING THEORY.

OBJECTIVE: The objective of this course is to equip the students withfundamental knowledge and problem solving skills in finite Fields, Field extension, polynomials with finite Fields, Coding scheme and Decoding scheme. It helps the students to master mathematical techniques and concepts used to analyse and understand the finite Fields and coding theory. The students will also learn to interpret the real-world meanings and implications of the mathematical results. Students learn todiscover and derive.

Learning Outcomes:

- This subject consists of two topics namely finite Fields and Coding theory.
- Students will also study the various methods of solving several types of problem related to finite Fields and Coding theory. Some applications for polynomials with finite Fields and coding are also included.
- Classify finite Field and Field extension according to type of the problem.
- Identify roots of irreducible polynomials over finite Field, roots of unity and Primitive roots of unity.
- To solve the problems related to Moebius function and Moebius Inversion Formula.
- The main purpose of the course is to introduce students to the theory and methods of finite Fields and Coding theory.

Course Title; Finite Fields & Coding theory Course Code

Duration of Examination: 3 hours Maximum-Marks: 100

Unit-1

- Field Extensions: Field, Prime Field, Algebraic extension, simple extension, minimal polynomial of an algebraic element, Finite extension, Transitivity of Finite extensions, Simple algebraic extension, Splitting field,

- Characterization of finite Fields: Finite fields, Number of elements in a finite field, Existence and uniqueness of finite fields, Subfields of a finite field

Unit-2

- Roots of an irreducible polynomials over finite fields: Nature of roots, Relation between splitting fields of two irreducible polynomials of same degree, Automorphisms of a finite field

- Trace: Definition and its basic properties, Relation between trace and linear transformations, Transitivity of Trace, Norm: Definition and its basic properties, Transitivity of Norm, Bases: Dual bases, normal basis, Artin lemma, Normal Basis Theorem,

Unit-3

- Roots of unity and Cyclotomic polynomials: Cyclotomic field, Primitive *nth* root of unity, Cyclotomic polynomial, Cyclotomic field as simple algebraic extension, Finite fields as Cyclotomic fields

- Representation of elements of finite fields: Some different ways of writing the elements of a finite field

- Irreducible Polynomials: Moebius function μ, Moebius Inversion formula, Number of monic irreducible polynomials of a given degree over a finite field, Product of all monic irreducible polynomials of a given degree over a finite field

Unit-4

- Linear Codes: Definition of Code Coding scheme and Decoding scheme, Linear Codes, Hamming distance and weight, *t*-error-correcting codes, Decoding of linear codes, Hamming Bound, Poltkin Bound, Gilbert-Varshamov Bound, Dual code

Unit-5

Cyclic Codes: Definition, Characterization of cyclic code in terms of an ideal, Generator polynomial of cyclic code, Parity-check polynomial of cyclic code, Relation between code polynomial and the roots of generator polynomial, BCH code, Minimum distance of BCH codes, Decoding algorithm for BCH codes

Textbooks:

1. R Lidl and H Niederreiter, Introduction to finite fields and Applications, Revised Edition, Cambridge University Press, 1992.

Reference books:

1. R Hill, A first course in Coding theory, Oxford Appl. Math and Comp. Sci. Series., Clarendon Press, Oxford, 1986.
2. R Lidl and H Niederreiter, Finite fields, Revised Edition, Cambridge University Press, 1997
3. Gary L. Mullen and C. Mummert, Finite fields and Applications, American Mathematical Society, Indian Edition 2012.

Kamlesh Kumar

Course Code: PGMAT3F006T

Course Title: Probability & Statistics

OBJECTIVES: The goal of the course is to acquaint students with various probability distributions as well as to improve their abilities and understanding of sampling distributions and hypothesis testing.

LEARNING OUTCOMES

Upon successful completion of this course, students will be:

- Able to comprehend the essential ideas of probability, such as random variables, event probability, additive rules, and conditional probability.
- able to comprehend Bayes' theorem notion
- Able to comprehend statistical ideas and measures at a basic level
- Able to construct the central limit theorem notion
- Understand Binomial, Geometrical, Negative Binomial, Pascal, Normal, and Exponential Distributions.
- Be able to comprehend the ideas of various parameter estimation approaches, such as the method of moments, maximum likelihood estimation, and confidence intervals.
- Possessing the ability to test hypotheses.

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Name of the course: **Probability and Statistics** **Course Credits: 4**

Course Code:

Unit-1

Review of probability theory including conditional probability, Some important theorems on probability, Exercise on probability and conditional probability, Independent Events, Total probability theorem and Bayes' Theorem. Addition and multiplication theorems of probability

Unit-2

Random Variables and Distribution function: discrete and continuous, Exercise on distribution functions, Two dimensional random variables: joint distribution function and marginal distribution, Expectation and moments about mean and origin, Covariance and conditional expectation and examples, Moment inequalities- Tchebyshef, Markov, Jensen, Moment generating function and characteristic function with their properties.

Unit-3

Standard discrete probability distributions: Discrete uniform distribution, Bernoulli distribution, Binomial distribution, Poisson distribution, Geometric distribution, Negative binomial distribution with their properties and examples. Some important theorems based on these distributions.

Unit-4

Standard continuous probability distributions: Continuous uniform distribution, Normal distribution, Exponential distribution, Gamma distribution or Erlang distribution, Weibull distribution, Triangular distribution, Standard Laplace (Double exponential) distribution, Cauchy distribution, with their properties and examples. Some important theorems based on these distributions.

Unit-5

More on two dimensional random variables: probability and distributions and examples. Transformation of random variables with examples, Central limit theorem and its applications, Large sample theory: types, parameter and statistics test of significance.

Reference books:

1. **S. C. Gupta and V. K. Kapoor**: Fundamentals of Mathematical Statistics, Sultan Chand and Sons, New Delhi.
2. **S. Palaniammal**, "Probability and Queueing Theory", PHI Learning Private Limited, Delhi.

Kamlesh Kumar Kamlesh Kumar
01-05-2019

Course Code: PGMAT3C006T

Course Title: DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES

OBJECTIVES: The goal of this course is to offer students with a foundation in differential geometry of curves and surfaces in space, with a focus on geometric aspects, as a foundation for further study or applications. Students will be introduced to the fundamental concepts of classical differential geometry before being shown how to apply characteristic classes, connections, and curvature tensors to Riemannian manifolds in detail.

LEARNING OUTCOMES:

Students would get familiar with basic notions and instruments of differential geometry, would enhance their methods of solving mathematical problems in various fields. Students would be capable of solving basic problems on differential geometry structures and objects on manifolds. The course introduces the fundamentals of differential geometry primarily by focussing on the theory of curves and surfaces in three space.

On completion of the course the student should have the following learning outcomes:

1. Define the equivalence of two curves.
2. Find the derivative map of an isometry.
3. Explain differential maps between surfaces and find derivatives of such maps.
4. Express definition and parameterization of surfaces.
5. Defines surfaces and their properties.
6. Express tangent spaces of surfaces.
7. Integrate differential forms on surfaces.

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Course code:

Course Title: Differential Geometry of Curves and Surfaces

Course Credits: 4

Unit-1

- Curves in the plane, arc length, reparametrization, plane curvature, Euler Theorem for plane curves; oriented curvature, Fundamental Theorem for curves in $\mathbb{R}^2$.
- Cuves in space, curvature and torsion, Serret-Frenet formulae, Fundamental Theorem for curves in $\mathbb{R}^3$.

Unit-2

- Surfaces in $\mathbb{R}^3$(2-manifolds), Regular surface, change of coordinates, Tangent plane, The first Fundamental form and its appications

Unit-3

- Normal curvature, principal curvatures, The Gauss map, Second Fundamental form, Gaussian curvature, Mean Curvature

Unit-4

- Equivalence of Surfaces, Isometries, Christoffel symbols, Theorema Egregium, Gauss Equations, Mainardi-Codazzi Equations, Fundamental Theorem for Regular Surfaces

Unit-5

- Geodesics: geodesic curvature, geodesic, Clairaut's Relation, exponental map, Hopf-Rinow Theorem, The Gauss-Bonnet Theorem.

Recommended Text

1. John McCleary, Geometry from a Diffentiable viewpoint, Second edition, Cambridge University Press, 2012.

References

1. A. Pressley, Elementary Differential Geometry, Springer, Indian Reprint, 2004
2. Manfredo P. do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, 1976.
3. D. J. Struik, Lectures on Differential Geometry, Dover, 1988.
4. Barett O'Neill, Elementary Differential Geometry Second edition, Academic Press(Elsevier), 2006.

Kamlesh Kumar

Course code:

Course title: Fields and Galois Theory

Course Credits: 4

Unit 1

- Field: Definition and Examples, Types of fields, algebraic and transcendental elements, minimal polynomial of an algebraic element, simple extension
- Degree of a field extension, multiplicative property of degree, Classification of Quadratic extensions,

Unit 2

- Straight edge and compass constructions: constructible numbers, degree of a constructible number, trisection of an angle, Construction of regular polygon
- Splitting Field of a Polynomial: Existence and Uniqueness, Multiple Roots, Perfect Field

Unit 3

- Galois group, Examples, Artin Lemma, Separable and Normal extensions, Galois extension, Fundamental Theorem of Galois Theory
- Fundamental Theorem on symmetric polynomials, Symmetric rational functions

Unit 4

- Solvability by radicals, Galois group of a polynomial, Cyclotomic field and its Galois group
- Galois group as permutation group of the roots

Unit 5

- The general equation of the n^{th} degree, Abel-Ruffini Theorem, Equations with rational co-efficients and Symmetric group as Galois group
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3. M Artin, Algebra, Second edition, PHI Learning Private Limited, New Delhi, 2012.
4. I N Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

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Advanced Measure Theory

UNIT-1

Topological Preliminaries: Urhyson's Lemma (Statement Only), Finite Partation of Unity, Definition of $C_C(X)$, Linear Functional on $C_C(X)$, Reisz Representation Theorem (Statement Only), Borel Measure, Definition and examples, Regularity and other properties of Borel Measures, Lusin's Theorem.

UNIT-2

L^P-spaces: Convex functions and inequalities, Jensen's inequality, Definition of L^P-spaces, Holder's inequality, Minkowski's inequality, Completeness of L^P-spaces. Simple functions, Denseness of $C_C(X)$ in L^P-spaces.

UNIT-3

Complex Measures: Definition and examples, Total variation of complex measures, Absolute countinity and Mutually singular. A theorem of Raydon and Nikodym, Hahn decomposition theorem.

UNIT-4

Differentiation of measure: Derivative of measure. Weak L^1 function, Lebeasgue points, Nicely shrinking sets and their properties. Fundamental theorem of calculus.

UNIT-5

Measurability of Product spaces: The functions f_x and f_y. The class Ω of sets in $s \times \tau$ and its properties. Definition of product measure. Fubini theorem, Examples of Fubini's theorem, Convolution of functions, Convolution theorem.

Books:

1. **Rudin, Walter**, Real & Complex analysis, ,McGraw-Hill,2000
2. **J.Yeh**. Real analysis: theory of Maesure and Integration 3rd Edition, World Scientific, 2000.

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