M A/M Sc Applied Mathematics, 2nd-Semester, 2016
End-Semester Examination

Course title: Partial Differential Equations
Course number: PGAMT2E005T
Time allowed: 3 hours
Maximum Marks: 100

Instructions for the candidates:

- *The question paper consist of three sections, namely,* **Section A**, **Section B** *and* **Section C**.
- *The* **section A** *consist of 10 objective type questions, and all the questions are compulsory in this section.*
- *The* **section B** *consist of 10 short answer type questions with 2 questions from each unit, and the candidate has to attempt 5 questions selecting one question from each unit.*
- *The* **section C** *consist of 5 long answer type questions, and the candidate has to attempt any 3 questions.*

Section A

1. The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation $\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$ 1.5
(a) linear 3rd order (b) nonlinear 3rd order (c) linear 1st order (d) nonlinear 1st order

2. A function which satisfies Laplace equation and possess first and second order partial derivatives, is known as 1.5
(a) Poisson equation (b) the spherical mean (c) wave equation (d) none of these

3. Using substitution, which of the following equation is a solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$? 1.5
(a) $\cos(3x - y)$ (b) $x^2 + y^2$ (c) $\sin(3x - 3y)$ (d) $e^{3\pi x}\sin(\pi y)$

4. A general method for finding the complete intergal or complete solution of a nonlinear PDE of first order of the form $f(x, y, z, p, q) = 0$. This is known as 1.5
(a) separable equation (b) Charpit's method (c) Clairaut's method (d) none of the these

5. If the Neumann problem for a bounded region has a solution then it is 1.5
(a) constant in $\mathbb{R}$ (b) unique (c) may not be unique (d) none of these

6. If $\delta(t)$ is a continuously differentiable. Dirac delta function vanishing for large t, then $\int_{-\infty}^{\infty} f(t)\delta(t)dt$ is equal to 1.5
(a) $f'(0)$ (b) $f'(a)$ (c) $-f'(0)$ (d) none of these

7. The complete intergal of $p(1 + q) = qz$ is
(a) $ln(az - 1) = x + ay + c$ (b) $ln(az - 1) = ax - by - c$ (c) $ln(az - 1) = ax + by + c$ (d) none of these 1.5

8. $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ is a
(a) nonlinear PDF of first order (b) linear PDF of first order (c) is alomst linear PDF of first order (d) none of these 1.5

9. The complete intergal of the equation $\sqrt{p} + \sqrt{q} = 1$ is
(a) $z = ax - (1 + \sqrt{a})^2 y + c$ (b) $z = ax + (1 + \sqrt{a})^2 y + c$ (c) $z = ax + (1 - \sqrt{a})^2 y + c$
(d) $z = ax - (1 - \sqrt{a})^2 y + c$ 1.5

10. Let $u = \psi(x, t)$ be the solution to the initial value problem $u_{tt} = u_{xx}$ for $-\infty < x < \infty$, $t > 0$ with $u(x, 0) = \sin x$, $u_t(x, 0) = \cos x$, then the value of $\psi(\frac{\pi}{2}, \frac{\pi}{6})$ is
(a) $\frac{\sqrt{3}}{2}$ (b) 1/2 (c) $1/\sqrt{2}$ (d) 1 1.5

Section B
Unit I

11. Find the characteristics of the PDE $p^2 + q^2 = 2$ and determine the intergal surface which passes through $x = 0$, $z = y$. 8

12. Define complete intergal. Find the complete intergal of the PDE $pqz = p^2(xq + p^2) + q^2(yp + q^2)$. 8

Unit - II

13. Find the complete intergal of the PDE $z^2 = pqxy$ by using Charpit Method. 8

14. Reduce the following equation to a canonical form and solve it: $yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$. 8

Unit - III

15. A thin rectangle homogenous thermally conducting plate lies in the xy-plane defined by $0 \leq x \leq a$, $0 \leq y \leq b$. The edge $y = 0$ is held at the temperature $Tx(x-a)$, where T is a constant, while the remaining edges are held at 0°C. The other faces are insulated and no internal sources and sinks are present. Find the steady state temperature inside the plate. 8

16. Define Laplace equation and show that $\psi = \frac{q}{|r-r'|}$, q is constant, is a solution of the Laplace equation. 8

Unit - IV

17. Define Dirac Delta function. Prove the following:
    a). $\int_{-\infty}^{\infty} \delta(t)dt = 1$;
    b). $\delta(-t) = \delta(t)$;
    c). $\delta(at) = \frac{1}{a}\delta(t)$, $a > 0$;
    d). if $\delta(t)$ is a continuously differentiable. Dirac delta function vanishing for large t, then

$$\int_{-\infty}^{\infty} f(t)\delta'(t)dt = -f'(0).$$

8

18. The ends A and B of a road, 10cm in length, are kept at temperature 0°C and 100°C until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20°C, and the end B is decreased to 60°C. Find the temperature distribution in the rod at time t. 8

Unit - V

19. Obtain the D'Alembert's solution of the one dimensional wave equation. 8

20. Solve one dimensional wave equation $u_{tt} = c^2 u_{xx}$, $0 \leq x \leq \pi$, $t \geq 0$ subject to $u = 0$ when $x = 0$ and $x = \pi$ and $u_t = 0$ when $t = 0$ and $u(x, 0) = x$, $0 < x < \pi$. 8

Section - C

21. Derive the characteristics equations of first order non-linear partial differential equations. 15

22. Define compatible systems of first order equations. Prove that the following PDE's $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution. 15

23. State and prove Dirichlet problem for a rectangle. 15

24. Define diffusion equation. Solve the one-dimentional diffusion equation in the region $0 \leq x \leq \pi$, $t > 0$, subject to the conditions:
    i) T remains finite as $t \to \infty$
    ii) $T = 0$, if $x = 0$ and π for all t
    iii) At $t = 0, T = \begin{cases} x & \text{if } 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} \leq x \leq \pi \end{cases}$ 15

25. State and prove uniqueness theorem for the wave equation. 15
