M A/M Sc Applied Mathematics
End-Semester Examination

Course Title: Optimization Techniques **Course code:** PGAMT2C004T
Time allowed: 3 hours **Maximum Marks:** 100

Instructions for the students-

- Attempt **all** the questions from section A.
- Solve **five questions** from section B, selecting one from each unit and **any three questions** from section C.
- For section A, each question carries **1.5 marks**. For section B, each question carries **8 marks**. For section C, each question carries **15 marks**.

Section-A

1. The method used to solve the assignment problem is called

(a) Reduced matrix method. (b) MODI method.
(c) Hungarian method. (d) All of these

2. The assignment problem

(a) requires that only one activity be assigned to each resource.
(b) is a special case of transportation problem.
(c) can be used to maximize resources. (d) all of the above.

3. If LPP has optimal solution at more than one vertex of the feasible region, then the LPP has

(a) infeasible solution. (b) no solution. (c) multiple solutions. (d) None of these

4. Feasible region of LPP is always

(a) bounded. (b) unbounded. (c) convex. (d) None of these.

5. One disadvantage of using North-West corner rule to find initial solution to the transportation problem is

(a) it is complicated to use. (b) it does not take into account the cost of transportation.
(c) it leads to a degenerate initial solution. (d) all of the above.

6. In an LPP with m constraints in n variables, the maximum numbers of basic solutions are

(a) $n_{C_{m+1}}$. (b) n_{C_m}. (c) $n_{C_{m-1}}$. (d) $n_{C_{m+2}}$.

7. If the primal has an unbounded solution, then its dual has

(a) optimal solution. (b) unbounded solution. (c) no-feasible solution. (d) multiple solution.

8. An assignment problem can be solved by

(a) simplex method. (b) transportation method.
(c) both (a) and (b). (d) none of these.

9. Branch and bound method divides the feasible solution space into smaller parts by

(a) branching. (b) bounding. (c) enumerating. (d) All of these.

10. Idle time on a machine means

(a) idle time of all the machines. (b) time a machine remains idle during elapsed.

(c) time of last job done on last machine. (d) None of these.

Section-B

Unit-I

1. Show that the set $S = \{(x_1 + x_2 + x_3): 2x_1 - x_2 + x_3 \leq 4\}$, is a convex set.

2. Solve the given LPP by Graphical method

$$Max\ Z = 4x_1 + 5x_2$$

subject to constraints are

$$x_1 + x_2 \geq 1; \quad -2x_1 + x_2 \leq 1; \quad 4x_1 - 2x_2 \leq 1 \quad \text{and} \quad x_1, x_2 \geq 0.$$

Unit-II

1. *Obtain an initial basic feasible solution to the following transportation problem using the VAM method*

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

where O_i and D_j denote i^{th} origin and j^{th} destination, respectively.

2. Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table:

2	9	2	7	1
6	8	7	6	1
4	6	5	3	1
4	2	7	3	1
5	3	9	5	1

Find the assignment of men to jobs that will minimize the total time taken.

Unit-III

1. Solve the travelling salesman problem given by the following table

∞	4	7	3	4
4	∞	6	3	4
7	6	∞	7	5
3	3	7	∞	7
4	4	5	7	∞

2. Find the sequence that minimizes the total elapsed time (in hours) required to complete the following jobs on three machines A, B, C and also calculate the idle time for each machine

	J_1	J_2	J_3	J_4	J_5	J_6
A	3	12	5	2	9	11
B	6	6	4	6	3	1
C	13	14	9	12	8	13

Unit-IV

1. Write an algorithm for Gomory's technique to solve the IPP.
2. Solve the following problem by Bellman's principle

$$Max\ Z = x_1.x_2.x_3$$
$$subject\ to\ constraints$$
$$x_1 + x_2 + x_3 = 5, \quad x_1, x_2, x_3 \geq 0.$$

Unit-V

1. Describe the Hessian bordered matrix method to solve the Non linear programming problem.
2. Define non-linear programming problem and write the Kuhn-Tucker conditions.

Section-C

1. Solve the following by using Two-phase method

$$Max\ Z = 3x_1 - x_2$$
$$subject\ to\ constraints$$
$$2x_1 + x_2 \geq 2,\ x_1 + 3x_2 \leq 2, \quad x_1, x_2 \geq 0.$$

2. A company has 4-warehouses and 6-stores; the cost of shipping one unit from warehouse i to store j is C_{ij}. If

$$C = [C_{ij}] = \begin{bmatrix} 7 & 10 & 7 & 4 & 7 & 8 \\ 5 & 1 & 5 & 5 & 3 & 3 \\ 4 & 3 & 7 & 9 & 1 & 9 \\ 4 & 6 & 9 & 0 & 0 & 8 \end{bmatrix},$$

and the requirements of the six stores are 4, 4, 6, 2, 4, 2 and quantities at the warehouse are 5, 6, 2, 9. Find the minimum cost solution for this *Transportation* problem.

3. Determine the replacement policy when the money value changes also it is assumed that the item has no resale value at the time of replacement and maintenance cost incurred at the beginning of year.

4. Solve the following problem by using Branch & Bound method

$$Max\ Z = x_1 + x_2$$
$$subject\ to\ constraints$$
$$3x_1 + 2x_2 \leq 12; \quad x_2 \leq 2; \quad x_1, x_2, \geq 0\ \ and\ integers.$$

5. Solve the following non-linear programming problem by constructing the Hessian bordered matrix

$$Mini\ Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$
$$\text{subject to constraints are}$$
$$x_1 + x_2 + x_3 = 15; \quad 2x_1 - x_2 + 2x_3 = 20.$$