SET-I

M A/ M Sc Applied Mathematics, Central University of Jammu

Semester-II, End Semester Examination 2016

Course Title: Complex Analysis — Course number: PGAMT2C003T

Time Allowed: 3 hours — Maximum Marks: 100

Instructions for the candidates:

- The question paper consist of three sections, namely, **Section A**, **Section B** and **Section C**.
- The **section A** consist of 10 objective type questions, and all the questions are compulsory in this section.
- The **section B** consist of 10 short answer type questions with 2 questions from each unit, and the candidate has to attempt 5 questions sèlecting one question from each unit.
- The **section C** consist of 5 long answer type questions, and the candidate has to attempt any 3 questions.

Section A

(1) Which of the following statement is correct statement ?

(a) $|z_1 z_2| = |z_1||z_2|$.

(b) $|z_1 z_2| \leq |z_1||z_2|$.

(c) $|z_1 z_2| \geq |z_1||z_2|$.

(d) None of the above. 1.5

(2) Consider the function $f(z) = \bar{z}$. Then

(a) $f(z)$ is non-analytic anywhere.

(b) $f(z)$ is analytic everywhere.

(c) $f(z)$ is analytic at $z = 1$ only.

(d) None of the above. 1.5

(3) Let $f(z)$ be an analytic function in a simple connected region R. If a and z are two points in R and $F(z) = \int_a^z f(z)dz$, then

(a) $F(z)$ is analytic in R and $F'(z) = f(z)$.

(b) $F(z)$ may not be analytic in R and $F'(z) = f(z)$.

(c) $F(z)$ is analytic in R and $F'(z) \neq f(z)$.

(d) None of the above. 1.5

(4) Consider the following two statements:

(i) If $f(z)$ is analytic in a region R and on its boundary C. Then $\int_C f(z)dz = 0$

(ii) If $f(z)$ is continuous in a simply connected region R and $\oint_C f(z)dz = 0$ then $f(z)$ is analytic in R, then

(a) Both (i) and (ii) are correct.

(b) Only (i) is correct.

(c) Only (ii) is correct.

(d) None of the above. 1.5

(5) If $f(z)$ is analytic within and on the boundary C of a simply connected region R, then

(a) $f'(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$.

(b) $f'(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^2}dz$.

(c) $f^n(a) = \frac{n!}{2\pi} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$.
(d) None of the above. 1.5

(6) The winding number of $|z-1| = 1$ with respect to $z = 3$ is
(a) 0.
(b) 1.
(c) 2.
(d) None of the above. 1.5

(7) The residue of $f(z) = \frac{z^3}{z^2-1}$ at $z = 1$ is
(a) 1.
(b) $\frac{1}{2}$.
(c) $\frac{1}{3}$.
(d) $\frac{1}{4}$. 1.5

(8) The radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}(z-1-i)^n$ is
(a) 0.
(b) ∞.
(c) 1
(d) n. 1.5

(9) Consider the following two statements
(i) The function $f(z) = e^z$ is conformal at every point in $\mathbb{C}$.
(ii) $g(z) = z^2$ is conformal at $z = 0$, then
(a) Both (i) and (ii) are true.
(b) Only (i) is true.
(c) Only (ii) is true.
(d) None of the above. 1.5

(10) Consider the function $f(z) = z^3 - 3z + 1$, then
(a) $f(z)$ is conformal at $z = \pm 1$.
(b) $f(z)$ is not conformal at $z = \pm 1$.
(c) $f(z)$ is not conformal at $z = 0$.
(d) None of the above. 1.5

Section B

Unit - I

(1) Define analytic functions and prove that $f(z) = e^{-\frac{1}{z^2}}$ has essential singularity at $z = 0$. 8

(2) Describe the stereographic projection with all necessary details. 8

Unit - II

(3) State and prove fundamental theorem of algebra. 8

(4) If $f(z)$ is analytic in a simply connected region R. Prove that $\int_a^b f(z)dz$ is independent of the path in R joining any two points a and b in R. 8

Unit - III

(5) State and prove Morera's theorem. 8

(6) State and prove Liouville's theorem. 8

Unit - IV

(7) Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 \cdot 4^n}$. 8

(8) State and prove Jordan's lemma. 8

Unit - V

(9) Find a suitable bilinear transformation which maps $z = 0, 1, -i$ into $w = 1, -1, i$ respectively. 8

(10) State and prove Schwartz's lemma. 8

Section - C

(11) Obtain an expression for Cauchy-Riemann equations in polar form. 15

(12) State and prove Green's theorem for complex valued function. 15

(13) State and prove Cauchy's integral formula for higher order derivatives. 15

(14) Prove that

$$\int_0^\infty \frac{x^3}{x^4 + a^4} \sin mx\,dx = \frac{\pi}{2} e^{\frac{-ma}{\sqrt{2}}} \cos \frac{ma}{\sqrt{2}}.$$

15

(15) Prove that the mapping $w = \frac{1}{z}$ transforms circle and straight lines into circle and straight lines. 15