SET-I

M A/ M Sc Applied Mathematics, Central University of Jammu
Semester-II, End Semester Examination 2016

Course Title: Topology
Time Allowed: 3 hours

Course number: PGAMT2C001T
Maximum Marks: 100

Instructions for the candidates:

- The question paper consist of three sections, namely, **Section A**, **Section B** and **Section C**.
- The **section A** consist of 10 objective type questions, and all the questions are compulsory in this section.
- The **section B** consist of 10 short answer type questions with 2 questions from each unit, and the candidate has to attempt 5 questions selecting one question from each unit.
- The **section C** consist of 5 long answer type questions, and the candidate has to attempt any 3 questions.

Section A

(1) Let $X = \{a, b, c\}$, then which of the following set is not a base for any topology whatsoever on X
(a) $\{\{a\}, \{b, c\}\}$.
(b) $\{\{a, b\}, \{b, c\}\}$.
(c) $\{\{a\}, \{b, \}\{c\}\}$. 1.5
(d) $\{\{a, b, c\}\}$.

(2) Limit point of a subset $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\}$ of R is
(a) 1.
(b) ∞.
(c) 0 1.5
(d) None of the above.

(3) The closure of the set $A = \{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, ...\}$ with respect to usual topology on R is
(a) $\bar{A} = \{1, 2, 3, 4, ...\}$.
(b) $\bar{A} = \{1, 2, \frac{3}{2}, \frac{4}{2}, ...\}$. 1.5
(c) $\bar{A} = \{1\}$.
(d) $\bar{A} = \phi$.

(4) Boundary of a set of integers $\mathbb{Z}$ is
(a) $\mathbb{N}$.
(b) $\mathbb{R}$. 1.5
(c) $\mathbb{Q}$.
(d) $\mathbb{R}^+$.

(5) Let (X, τ) be a topological space, then
(a) $\text{ext}(X) = \phi$.
(b) $\text{ext}(X) = X$. 1.5
(c) $\text{ext}(X) \neq X$.
(d) None of the above.

(6) Consider the following statements
(i) The cantor set is open subset of $\mathbb{R}$
(ii) The cantor set is countable. Then

(a) Both (i) and (ii) are true.
(b) Both (i) and (ii) are false.
(c) Only (i) is true. 1.5
(d) Only (ii) is true.

(7) Consider the following statements
(i) The product of a finite number of Housdorff spaces is not Housdorff space.
(ii) If X is connected then every quotient space of X is not connected. Then
(a) Both (i) and (ii) are true.
(b) Both (i) and (ii) are false.
(c) Only (i) is true. 1.5
(d) Only (ii) is true.

(8) Consider the following statements
(i) The product of T_i-spaces is not a T_i space
(ii) Every T_2 space is not T_1. Then
(a) Both (i) and (ii) are true.
(b) Both (i) and (ii) are false.
(c) Only (i) is true. 1.5
(d) Only (ii) is true.

(9) Consider the following two statements
(i) Every regular Lindelof space is not normal.
(ii) Every completely normal space is not normal.
(a) Both (i) and (ii) are true.
(b) Both (i) and (ii) are false.
(c) Only (i) is true. 1.5
(d) Only (ii) is true.

(10) Consider the following two statements
(i) Every compact Housdorff space is not normal.
(ii) Every metric space is not normal. Then
(a) Both (i) and (ii) are true.
(b) Both (i) and (ii) are false.
(c) Only (i) is true. 1.5
(d) Only (ii) is true.

Section B

Unit - I

(1) Define a topology on a non-empty set A (say). If $|A| = 3$, then determine/construct all the topologies on set A. 8
(2) Show that a set A is closed if and only if A contains all of its points. 8

Unit - II

(3) Show that a topology on a finite set is compact. 8
(4) Show that the closure of a connected set is connected. 8

Unit - III

(5) Show that the continuous image of a compact topological space is compact. 8
(6) Describe the cantor set with all necessary detail. Show that $\frac{1}{36}$ is a member of the cantor set. 8

Unit - IV

(7) Let $f : Y \to X$ be a function from a space Y into a product space $X = \prod_{i=1}^{n} X_i$. Then prove that f is continuous if and only if the composition $p_i f$ of f with each projection map is continuous. 8

(8) State and prove Tychonoff theorem. 8

Unit - V

(9) Prove that a T_1-space X is regular if and only if each point a in X and each open set U containing a, there is an open set W containing a whose closure is contained in U. 8

(10) Prove that every regular Lindelöf space is normal. 8

Section - C

(11) Show that $\bar{A} = A \cup D(A)$, where A is any subset of topological space (X, τ) 15

(12) Show that a topological space (X, τ) is disconnected if and only if $\exists$ a continuous function from X onto two points discrete space. 15

(13) Let (X, τ) be any topological space, $(Y, \tau/_Y)$ be a subspace of (X, τ) and $A \subseteq Y$, then A is compact relative to X if and only if A is compact relative to Y. 15

(14) Prove that the product of a finite number of compact spaces is compact. 15

(15) State and prove Tietze extension theorem. 15