Department of Mathematics, Central University of Jammu
End Semester Examination, 4th Semester Examination 2016

Course Title: Introductory Analysis
Time Allowed: 3 hours

Course number: MAMT-475
Maximum Marks: 100

Instructions for the candidates:

- The question paper consist of three sections, namely, **Section A**, **Section B** and **Section C**.
- The **section A** consist of *10 objective type* questions, and all the questions are compulsory in this section.
- The **section B** consist of 8 short answer type questions, and the candidate has to attempt any 5 questions.
- The **section C** consist of 10 long answer type questions with 2 questions from each unit, and the candidate has to attempt 5 questions selecting one question from each unit.

Section A

1. Suppose $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ are having their usual meaning. A real sequence is a function from 1
   (a) $\mathbb{N}$ to $\mathbb{R}$.
   (b) $\mathbb{N}$ to $\mathbb{C}$.
   (c) $\mathbb{R}$ to $\mathbb{C}$.
   (d) None of the above.
2. If there exists a real number K such that $S_n \geq K\ \forall\ n \in \mathbb{N}$ for a sequence $< S_n >$, then 1
   (a) $< S_n >$ is bounded
   (b) $< S_n >$ is bounded below.
   (c) $< S_n >$ is bounded above.
   (d)None of the above.
3. If $S = \{1, 2, 3, 4, 5\}$, then one of the lower bound of S is 1
   (a) 2.
   (b) 0.
   (c) 5.
   (d)None of the above.
4. The set of real numbers $\mathbb{R}$ is 1
   (a) countable set.
   (b) countably infinite.
   (c) uncountable.
   (d) None of the above.
5. The following statement:
   "The set of real numbers is the union of set of rational and irrational numbers" is 1
   (a) true.
   (b) may or may not.
   (c) false.
   (d) None of the above.
6. Every absolutely convergent series is 1
   (a) convergent.
   (b) oscillatory.
   (c) divergent.
   (d) None of the above.
7. The right hand limit of $f(x) = \frac{1}{1+e^{-1/x}}$ is 1
   (a) 0. (b) 1. (c) ∞. (d) None of the above.
8. The function $f(x) = \sin x,\ x \in [0, \infty)$ is 1
   (a) uniformly continuous.
   (b) limit does not exists at $x = 0$.
   (c) not uniformly continuous.
   (d) None of the above.

9. The left hand derivative of a function $f(x)$ at $x = a$ is given by

(a) $\lim_{x\to a-0} \frac{f(x)-f(a)}{x-a}$.

(b) $\lim_{x\to 0} \frac{f(x)-f(a)}{x-a}$.

(c) $\lim_{x\to a+0} \frac{f(x)-f(a)}{x-a}$.

(d) None of the above. 1

10. Consider the following two statements:

(i) If two functions have equal derivative at all the points of (a, b) *then they differ by a constant.*

(ii) $\frac{\tan x}{x} > \frac{x}{\sin x}$, $0 < x < \frac{\pi}{2}$

(a) only (i) is true.

(b) only (ii) is true.

(c) Both (i) and (ii) are true.

(d) Both (i) and (ii) are false. 1

Section B

1. Show that a function which is uniformly continuous on an interval is continuous on that interval. 6

2. Show that $\lim_{x\to 0} x \sin\frac{1}{x} = 0$ 6

3. State and prove Darboux's theorem. 6

4. Prove that a function which is differentiable at a point is necessarily continuous at that point. 6

5. Give the definition of bounded sequence with at least one example . 6

6. Prove that every convergent sequence is bounded. 6

7. Give the definition and at least one example of limit points of a sequence. 6

8. By using logarithmic test, test the convergence of the series

$$1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \cdots, \text{ for } x > 0.$$

6

Section - C

Unit - I

9. Define supremum and infimum for a set. Determine supremum and infimum of the set $\{4 + \frac{1}{n} : n \in \mathbb{N}\}$. 12

10. Give all the details of field structure of real numbers. 12

Unit - II

11. If $\lim_{n\to\infty} a_n = \ell$, then prove that

$$\lim_{n\to\infty} \frac{a_1 + a_2 + \cdots + a_n}{n} = \ell.$$

12

12. State and prove the necessary and sufficient condition for the convergence of a sequence. 12

Unit - III

13. Show that the series

$$\sum \frac{3 \cdot 6 \cdot 9 ... 3n}{7 \cdot 10 \cdot 13 ... (3n+4)} x^n, \; x > 0$$

converges for $x \leq 1$, and diverges for $x > 1$. 12

14. Using D'Alembert's ratio test, test the convergence of the series $\sum \frac{n^2-1}{n^2+1} x^n, \; x > 0$ 12

Unit - IV

15. Define uniform continuity. Prove that the function $f(x) = \sin x$ is uniformly continuous on $[0, \infty)$. 12

16. If a function f is continuous at an interior point c of an interval $[a, b]$ and $f(c) \neq 0$ then there exists a $\delta > 0$ such that $f(x)$ has the same sign as $f(c)$ for every $x \in (c - \delta, c + \delta)$ 12

Unit - V

17. State and prove Rolle's theorem. 12

18. State and prove Lagrange's mean value theorem. 12