M A/M Sc Applied Mathematics, 4th-Semester, 2016
End-Semester Examination

Course title: Wavelet Analysis and Applications
Time allowed: 3 hours
Instructions for the candidates:

Course number: *MAMT 419*
Maximum Marks: *100*

- *The question paper consist of three sections, namely,* **Section A**, **Section B** *and* **Section C.**
- *The* **section A** *consist of 10 objective type questions, and all the questions are compulsory in this section.*
- *The* **section B** *consist of 8 short answer type questions, and the candidate has to attempt any 5 questions.*
- *The* **section C** *consist of 10 long answer type questions with 5 questions from each unit, and the candidate has to attempt 2 questions selecting one question from each unit.*

Section A

1. For $f, g \in L_1(\mathbb{Z}_N)$, the DFT $D(f*g)(x)$ is equal to
   (a) $Df(x).g(x) + f(x).Dg(x)$ (b) $Df(x).Dg(x)$ (c) $Df(x)*Dg(x)$ (d) none of these 1

2. Which of the following is false?
   (a) $f \star g = g \star f$ (b) $f \star (g \star h) = (f \star g) \star h$ (c) $(f \star g) + h = f \star h + g \star h$ (d) $f \star (g+h) = f \star g + f \star h$ 1

3. If $F : L_2(\mathbb{R}) \to L_2(\mathbb{R})$ is a Fourier transform, then F is
   (a) isomorphism (b) discontinuous (c) non-linear (d) none of these 1

4. If p: every tight frame is exact, q: every exact frame is tight, r: every tight frame is linearly independent, then which of the following is true?
   (a) p (b) q (c) r (d) none of these 1

5. Let $\{x_n\}$ be a frame with frame bonds A and B. Which of the following is true?

   (a) $A||X||^2 = \sum_{n\in N} |\langle x, x_n\rangle|^2 = B||x||^2 \forall x \in H$. (b) $A||X||^2 < \sum_{n\in N} |\langle x, x_n\rangle|^2 < B||x||^2 \forall x \in H$. (c) $A||X||^2 \leq \sum_{n\in N} |\langle x, x_n\rangle|^2 \leq B||x||^2 \forall x \in H$. (d) none of these 1

6. If $N = RC$, the number of multiplications required to compute $Df(n)$ is equal to
   (a) $NR + C$ (b) $R + NC$ (c) $NR + RC$ (d) $NR + NC$ 1

7. Let $((V_n), \varphi)$ be an MRA with mother wavelet ψ. Which of the following is true?
   (a) $\{\psi(t-n) : n \in \mathbb{Z}\}$ is an orthonormal basis for V_o (b) $\{\psi(t-n) : n \in \mathbb{Z}\}$ is an orthonormal basis for W_o
   (c) $\{\psi(t-n) : n \in \mathbb{Z}\}$ is an orthonormal basis for V_1 (d) none of the above 1

8. Let $\hat{f}$ be the L_2-Fourier transform of $f \in L_2(\mathbb{R})$. Then $\{f(t-n) : n \in \mathbb{Z}\}$ is an orthonormal family iff
   (a) $\sum_{n\in\mathbb{Z}} |\hat{f}(\omega + 2n\pi)| = 1$ (b) $\sum_{n\in\mathbb{Z}} |\hat{f}(\omega + 2n\pi)|^{\frac{1}{2}} = 1$ (c) $\sum_{n\in\mathbb{Z}} |\hat{f}(\omega + 2n\pi)|^2 = 1$ (d) none of the above 1

9. Which of the following is the scaling identity?
   (a) $|m_\varphi(\omega)| + |m_\varphi(\omega + \pi)| = 1$ *a.e.* (b) $|m_\varphi(\omega)| - |m_\varphi(\omega+\pi)| = 1$ *a.e.* (c) $|m_\varphi(\omega)|^2 + |m_\varphi(\omega+\pi)|^2 = 1$ *a.e.*
   (d) $|m_\varphi(\omega)|^2 - |m_\varphi(\omega+\pi)|^2 = 1a.e.$ 1

10. Which of the following does not have compact support?
    (a)$\chi_{[0,1]}$ (b) $\chi_{[0,2]}$ (c) $\chi_{[0,\infty)}$ (d)none of the above 1

Section B

11. Let (V_n, ϕ) be an MRA and $g \in V_1$. Prove that $\hat{g}(2w) = m_g(w)\hat{\phi}(w)$ *a.e* 6

12. Define DFT map $F : L_1(\mathbb{Z}_w) \to L_1(\mathbb{Z}_N)$ and show that it is a linear bijection. 6

13. Let $f : \mathbb{Z}_N \to \mathbb{C}$ be a function and let $g(n) = f(n-j)$. Then show that $Dg(n) = Df(n)e^{-2\pi ijn}$. 6

14. Let $V_j = \{f \in L_2\mathbb{R} : f$ is constant on $[2^{-j}n, 2^{-j}(n+1)$ for $n \in \mathbb{Z}\}$ and $\varphi = \chi_{[0,1)}$. Then show that $\{2^{\frac{j}{2}}\varphi(2^jt - n) : n \in \mathbb{Z}\}$ form an orthonormal basis for V_j for any $j \in \mathbb{Z}$. 6

15. If φ is a sclaing function with compact support and $\varphi(0) \neq 0$, then show that φ is continuous 6

16. If $\{h(x-n) : n \in \mathbb{Z}\}$ is a Riesz basis for the closed subspace V_0 of $L_2\mathbb{R}$. Then show that there exists $\phi \in V_0$ such that $\{\phi(x-n) : n \in \mathbb{Z}\}$ is an orthonormal basis for V_0. 6

17. Define the transposition operator T_0 and show that if f is any 2π perodic continuous function, then

$$\int_{-\pi}^{\pi} T_0^n f(w)dw = \int_{-2^n\pi}^{2^n\pi} \prod_{j=1}^{n} |m_\varphi((2^{-j}w))^2| f(2^{-n}w)d\dot{w}$$

6

18. Write brief note on applications of wavelets to Medicines. 6

Section - C
Unit - I

19. State and prove Sampling Theorem. 12

20. For $N = 2^k$ and $f \in \mathbb{C}^N$, prove that the number of multiplications required to compute $Df(n)$ is $2N \log_2^N$. 12

Unit - II

21. State and prove representation theorem of filter m_g for $g \in W_o$. 12

22. State and Prove Mother Wavelet Theorem. 12

Unit - III

23. If φ is a scaling function having compact support and $\hat{\varphi}(0) \neq 0$ with

$$m_\varphi(\xi) = \sum_{k=-n}^{n} \frac{c_k}{\sqrt{2}} e^{-ik\xi} = 1,$$

then show that

$$\prod_{j \in \mathbb{N}} m_\varphi\left(\frac{\xi}{2^j}\right)$$

converges uniformly on bounded subsets of $\mathbb{R}$. 12

24. Establish that the trigonometric polynomials are not sufficient to generate wavelets.

Unit - IV

25. Write a detailed note on Bi-orthogonal wavelets. 12

26. Let $\{x_n\}$ be a sequence of vectors in Hilbert space H. Then prove that $\{x_n\}$ is a frame with frame bounds A and B if and only if the frame operator $S_x = \sum_{n \in \mathbb{N}} \langle x, x_n \rangle x_n$ is bounded with $AI \leq S \leq BI$ where $I : H \to H$ is the identity map. 12

Unit - V

27. Define Neural networks. Discuss the applications of wavelets in Neural networks. 12

28. Write a note on applications of wavelets to economics and finance. 12