M A/M Sc Applied Mathematics
End-Semester Examination

Course title: Applied Statistics **Course Code:** PGAMT3F005T

Maximum Marks: 100

Instructions for the students:

- Attempt **all the questions** from section A.
- Solve **one question** of each unit from section B and any **three questions** from section C.
- For section A, each question carries **1.5 marks**. For section B, each question carries **8 marks**. For section C, each question carries **15 marks**.

Section-A (Objectives)

1. If A and B are any two events, then

(a) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ (b) $P(A \cap \bar{B}) = P(B) - P(A \cap B)$
(c) $P(A \cap \bar{B}) = P(A) - P(A \cup B)$ (d) $P(A \cap \bar{B}) = P(B) - P(A \cup B)$

2. If events A and B are mutually exclusive, then P(AB) is

(a) 1 (b) 0 (c) 0.50 (d) 0.25

3. If F(x) be a distribution function of a random variable X, then F(x) is

(a) a bounded function (b) a monotonically non-decreasing function
(c) always a continuous function (d) all of these.

4. In Markov chain, a state-i is non-null-persistent, if

(a) $\mu_{ii} = \infty$ (b) $\mu_{ii} < \infty$ (c) $\mu_{ii} = 0$ (d) $\mu_{ii} = 1$.

5. The following distribution(s) function has *memory less property*

(a) Binomial distribution (b) Poisson distribution
(c) Exponential distribution (d) All of these.

6. If $X \sim Poisson(m)$ then, the Var(X) is given by

(a) $\frac{1}{m}$ (b) $\frac{1}{m^2}$ (c) 1 (d) none of these.

7. Poisson process is

(a) a stationary process (b) not a stationary process
(c) a covariance stationary process (d) none of these.

8. The normal curve is symmetrical about

(a) the x-axis (b) the line x = 0 (c) the line x = μ(mean) (d) the line y = μ(mean).

9. If $P(A) = 0.5,\ P(B) = 0.3$ and $P(A \cap B) = 0.15$, then $P(A/\bar{B})$ is

(a) 0.3 (b) 0.4 (c) 0.5 (d) 0.6

10. In M/M/1 queueing model, P_0 is

(a) $(1-\rho)$. (b) $(1-\rho^2)$. (c) ρ. (d) $\frac{1}{1-\rho}$.

Section-B

Unit-1

1 Let X and Y be integer valued random variables with $P(X=m, Y=n) = q^2 p^{m+n-2},\ m,n = 1,2,3,\ldots$ and $p+q=1$. Are X and Y are independent?

2 Find the MGF of the random variable X whose probability function $P(X=x) = \frac{1}{2^x};\ x = 1,2,3,\ldots$ Also find its mean.

Unit-2

1. If X and Y be independent uniform random variables over (0, 1), find the pdf of Z = X+Y.
2. If X is a Poisson random variables with $P(X=1) = P(X=2)$, find $P(X \geq 3)$.

Unit-3

1. Show that a Poisson process is a Markov process.
2. Show that the difference of two independent Poisson processes needs not to be a Poisson process.

Unit-4

1. Find the solution of an LPP by Graphical method

$$\begin{aligned} \text{Min}\, z &= x_1 + 5x_2 \quad s.t. \\ 4x_1 + x_2 &\geq 10 \\ x_1 - 2x_2 &= 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2 Find the dual of the following LPP

$$\text{Min}\, z = 2x_1 - 3x_2 + x_3 \quad s.t.$$

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= 3 \\ 2x_1 - 2x_2 &\leq 1 \\ 7x_1 - x_3 &\geq 2 \end{aligned}$$

$x_1, x_2 \geq 0, x_3$ is unrestricted in sign.

Unit-5

1. If X(t) denotes the number of events that takes place in an interval of length t, then under the axioms of Poisson process, show that the distribution of X(t) is Poisson with parameter λt.
2. Find the expected length of non-empty queue for the M/M/S queueing system with infinite capacity.

Section-C

1. Two discrete random variables X and Y have the joint probability density function $P(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!},\ y = 0,1,\ldots,x;\ x = 0,1,\ldots$ where λ and $0 < p < 1$ are constants. Find

(i) The marginal probability density function of X and Y.

(ii) The conditional distribution of Y for a given X and of X for a given Y.

2. If X and Y are two independent random variables each following Normal distribution with mean 0 and variance 4. Find the pdf of Z = X+Y.

3. Given the TPM with 3- states (1, 2, 3) of the Markov chain $\{X_n(t)\}$ as:

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$ and the initial distribution is $P^{(0)} = [0.7,\ 0.2,\ 0.1]$.

Find (i) $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

4. Solve the LPP by using the Big-M method

$$\text{Max}\, z = 15x_1 + 25\, x_2 \quad s.t.$$

$$7x_1 + 6x_2 \geq 20,\ \ 8x_1 + 5x_2 \leq 30\ ,\ 3x_1 - 2x_2 \leq 18 \text{ and } x_1, x_2 \geq 0$$

5. Derive the formula for steady-state probability distribution and waiting time distribution for finite capacity queueing system with single server. Also, calculate the expected system size.