M A/ M Sc Applied Mathematics, 3rd Semester Examination 2016
End Semester Examination

Course Title: Basic Linear Algebra | **I.D. Course Number**: PGAM3IOO1T-
Time Allowed: 3 hours | **Maximum Marks**: 100

Instructions for the candidates:

- The question paper consist of three sections, namely, **Section A**, **Section B** and **Section C**.
- The **section A** consist of 10 objective type questions, and all the questions are compulsory in this section.
- The **section B** consist of 10 short answer type questions with 2 questions from each unit, and the candidate has to attempt 5 questions selecting one question from each unit.
- The **section C** consist of 5 long answer type questions, and the candidate has to attempt any 3 questions.

Section A

1. A matrix A is said to be orthogonal if

   (a) $A^T = -A$. (b) $A^T = A$. (c) $A^T = A^{-1}$. (d) None of the above. 1.5

2. If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$. Then which of the following is correct?

   (a) $|A| = a_{11}a_{22} - a_{12}a_{21}$.
   (b) $|A| = a_{12}a_{21} - a_{11}a_{22}$.
   (c) $|A| = a_{11}a_{22} + a_{12}a_{21}$.
   (d)None of the above. 1.5

3. Let A be a square matrix and A has a row (or columns) of zeros. Then

   (a) $|A| = 0$.
   (b) $|A|$ may or may not be zero.
   (c) $|A| \neq 0$.
   (d) None of the above. 1.5

4. Consider the following two statements:

   (i) Every non zero element of a field $\mathbb{F}$ possess multiplicative inverse.
   (ii) There exists $1 \in \mathbb{F}$ such that $1.a = a$ for all $a \in \mathbb{F}$. Then

   (a) Only (i) is true.
   (b) Both (i) and (ii) are true.
   (c) Only (ii) is true.
   (d) None of the above. 1.5

5. The standard basis of $V_2(R)$ is

   (a) $B = \{(1,0),(0,1)\}$.
   (b) $B = \{(1,2),(0,3)\}$.
   (c) $B = \{(1,1),(0,1)\}$.
   (d) Both (a) and (b) are false. 1.5

6. The dimension m of a subspace of a vector space $V(F)$ of dimension n satisfy

   (a) $m \leq n$. (b) $m > n$. (c) $\gcd(m,n) = 1$. (d) None of the above. 1.5

7. The eigen values of the matrix $A = \begin{pmatrix} i & 0 \\ 2 & 6i \end{pmatrix}$ are

   (a) $i, 6i$. (b) $5i, 9$. (c) $3, 7i$. (d) None of the above. 1.5

8. Let U and V be two vector spaces over the field F. Then for all $\alpha, \beta \in V$ and $a, b \in F$, a linear transformation T from U to V is

(a) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$.

(b) $T(a\alpha + b\beta) = (a+b)T(\alpha) + (a+b)T(\beta)$.

(c) $T(a\alpha + b\beta) = \alpha T(a) + \beta T(b)$.

(d) None of the above. 1.5

9. The rank of the matrix $A = \begin{pmatrix} 1 & 2 \\ i & 2i \end{pmatrix}$ is

(a) 1. (b) 2. (c) 3. (d) None of the above. 1.5

10. Which of the following is an orthonormal set

(a) $S = \{(1,0,0),(0,1,0),(0,0,1)\}$.

(b) $S = \{(1,2,3),(0,3,4),(0,0,5)\}$.

(c) $S = \{(1,1,1),(1,1,0),(0,0,1)\}$.

(d) None of the above. 1.5

Section B

Unit - I

1. Write a system of n-linear equations and hence define coefficient and augmented matrices. 8

2. Check whether the property $A^\theta A = A$ holds for the matrix $A = \begin{pmatrix} i & 1+i \\ 2 & -i \end{pmatrix}$. 8

Unit - II

3. For the following system of equations:

$2x - 5y + 2z = 2,$

$x + 2y - 4z = 5,$

$3x - 4y - 6z = 1.$

Compute the value of determinant of the coefficient matrix. 8

4. Find the value of the determinant of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & -2 \\ 1 & -3 & 4 \end{pmatrix}.$$

8

Unit - III

5. Determine the rank of the matrix $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 1 \\ 3 & 2 & 5 \end{pmatrix}$ by reducing into Echelon form. 8

6. Let R be the field of real numbers. Show that the following is subspace of $V_3(R)$

$$W = \{(a,b,c) : a,b,c \in R\}.$$

8

Unit - IV

7. Consider the following linear operator T on R^2 defined by $T(x, y) = (2x - 7y, 4x + 3y)$ with respect to the standard basis $S = \{(1,0), (0,1)\}$.

(i) Find the matrix A of T relative to S.

(ii) Find determinant and trace of matrix A. 8

8. Prove that the function $T(a, b) = (a + b, a - b, b)$ is a linear transformation on R^2. 8

Unit - V

9. Find the matrix of the linear operator on R^3 defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$ with respect to the standard basis of R^3. Find the eigen values and eigen vectors of T by computing characteristic polynomial. 8

10. Consider the vectors $u = (1, 2, 4)$, $v = (2, -3, 5)$ and $w = (4, 2, -3)$ in R^3. Find $u.v$, $||u||$, $||v||$ and $(u + v).w$. 8

Section - C

11. Define the following and produce atleast one example supporting to the claim.

(i) Symmetric matrix.

(ii) Upper triangular matrix.

(iii) Lower triangular matrix. 5+5+5

12. Check whether the property $|AB| = |A||B|$ holds for the matrices

$$A = \begin{pmatrix} 2 & 6 & 1 \\ 4 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 5 & 1 & 2 \\ 3 & -2 & 1 \\ 1 & 2 & -1 \end{pmatrix}$$ 15

13. Prove that an ordered n-tuples over the field $\mathbb{F}$ of real numbers forms a vector space. 15

14. Show that the function $T(a, b) = (a + b, a - b)$ is a linear transformation. Compute range, rank, null space and nullity of T. 15

15. Define Gram Schmidt orthogonalization process. Obtain an orthonormal basis of T with respect to the standard inner product in R^3 generated by $S = \{(1, 0, 3), (2, 1, 1), (0, 2, 1)\}$. 15