M A/M Sc Applied Mathematics, 3rd Semester, 2016
End-Semester Examination

Course title: Applied Statistics **Course number:** MAMT-305

Time allowed: 3 hours **Maximum Marks:** 100

Instructions for the students:

- Attempt all the questions from section A.
- Solve any **five questions** from section B and **one question** of each unit from section C.
- For section A, each question carries **one mark**. For section B, each question carries **six marks**. For section C, each question carries **twelve marks**.

Section-A (Objectives)

1. If A and B are any two events, then

(a) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ (b) $P(A \cap \bar{B}) = P(B) - P(A \cap B)$

(c) $P(A \cap \bar{B}) = P(A) - P(A \cup B)$ (d) $P(A \cap \bar{B}) = P(B) - P(A \cup B)$

2. If events A and B are mutually exclusive, then P(AB) is

(a) 1 (b) 0 (c) 0.50 (d) 0.25

3. If F(x) be a cumulative distribution function (C.D.F.) of random variable X, then F(x) is

(a) a bounded function (b) an unbounded function (c) may or may not be bounded (d) none of these.

4. In Markov chain, a state-i is null-persistent, if

(a) $\mu_{ii} = \infty$ (b) $\mu_{ii} < \infty$ (c) $\mu_{ii} = 0$ (d) $\mu_{ii} = 1$.

5. The following distribution(s) function has *memory less property*

(a) Binomial distribution (b) Poisson distribution

(c) Exponential distribution (d) All of these.

6. If $X \sim Poisson(m)$ then, the Var(X) is given by

(a) $\frac{1}{m}$ (b) $\frac{1}{m^2}$ (c) 1 (d) none of these.

7. If a Markov chain is ergodic, then

(a) chain is null-persistent (b) chain is periodic with period more than 2

(c) chain is aperiodic (d) none of these.

8. The normal curve is symmetrical about

(a) the x-axis (b) the line x = 0 (c) the line x = μ(mean) (d) the line y = μ(mean).

9. The expectation of a bounded random variable –

(a) always exists. (b) may or may not exist. (c) exists infinitely. (d) does not exist.

10. In M/M/1 queueing model, P_0 is

(a) $(1-\rho)$. (b) $(1-\rho^2)$. (c) ρ. (d) $\frac{1}{1-\rho}$.

Section-B

1. Let X and Y be integer valued random variables with $P(X=m, Y=n) = q^2p^{m+n-2},\ m,n = 1,2,3,....$ and $p+q=1$. Are X and Y are independent?
2. If X and Y are independent random variables then, show that $Var(X+Y) = Var(X) + Var(Y)$.
3. Show that the difference of two independent Poisson processes needs not to be a Poisson process.
4. Consider the TPM with states (0, 1, 2) as: $P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/2 \\ 0 & 3/4 & 1/4 \end{bmatrix}$;

$$Prob(X_0 = i) = 1/3;\ i = 0,1,2. \text{ Then,}$$

(a) Find $p_{01}^{(2)}$

(b) $Prob[X_2 = 1, X_0 = 0]$

5. Consider a stochastic process as $X(t) = A cos\omega t + B sin\omega t$, where A and B are uncorrelated random variables with mean= 0 and variance = 1. Then, show that the process is covariance stationary process.
6. Find the dual of an LPP

$$\begin{aligned} \text{Min } z = x_1 + 5x_2 + x_3 \quad s.t. \\ 4x_1 + x_2 + x_3 = 6 \\ x_1 - 2x_2 + x_3 = 4 \\ x_1, x_2, x_3 \geq 0 \end{aligned}$$

7. For the Markovian properties of inter-arrival time, if there is no arrival till time t_0, then the probability that first arrival occurs before time t_1 is same the probability that first arrival occurs before time $(t_1 - t_0)$ $i.e.$

$$Prob[T \leq t_1/T > t_0] = Prob[0 \leq T \leq t_1 - t_0]$$

8. Find the probability of no-customer in the system for the M/M/2 queueing model.

Section-C

Unit-1

1. Two discrete random variables X and Y have the joint probability density function $P(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}$, $y = 0,1,...,x;\ x = 0,1,....$ where λ and $0 < p < 1$ are constants. Find

(i) The marginal probability density function of X and Y.

(ii) The conditional distribution of Y for a given X and of X for a given Y.

2. A two-tetrahedral (a dice having 4-sides) with sides numbered as 1,2,3,4, are tossed. Let X-denotes the number on the downturn face of the first tetrahedron and Y-denotes the maximum of the downturned faces of both dice. Find the joint probability density function, marginal density function of X *and* Y, conditional distribution function of X and Y.

Unit-2

1. Find the mean and variance for Binomial distribution. If X and Y are independent Poisson variates, then show that the conditional distribution probability of X given X+Y, is Binomial.
2. Find the moment generating function (MGF) of normal distribution and using MGF, calculate the mean and variance of normal distribution.

Unit-3

1. State and prove the steady-state condition for the Markov chain.
2. Given the TPM with 4- states (1, 2, 3, 4) of the Markov chain as:

$$P = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$; Examine the nature of states in the respect of transient, null persistent or non- null persistent, ergodic.

Unit-4

1. Solve the LPP by using the Big-M method

$$\text{Max } z = 15x_1 + 25x_2 \quad s.t.$$

$$7x_1 + 6x_2 \geq 20, \; 8x_1 + 5x_2 \leq 30, \; 3x_1 - 2x_2 = 18 \text{ and } x_1, x_2 \geq 0$$

2. Find the dual of the following LPP

$$\text{Min } z = x_1 + x_2 + x_3 \quad s.t.$$

$$x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_2 \leq 3$$
$$2x_1 - x_3 \geq 4$$
$$x_1, x_2 \geq 0, x_3 \text{ is unrestricted in sign.}$$

Unit-5

1. If X(t) denotes the number of events that takes place in an interval of length t, then under the axioms of Poisson process, show that the distribution of X(t) is Poisson with parameter λt.
2. Derive the formula for steady-state probability distribution for M/M/1/K queueing model. Also, calculate the expected system size.