Central University of Jammu, Jammu
Department of Mathematics

Name of the course: **Some Stochastic Models**
Course Code: **PGAMT4E004T**

Max. Marks: 100

Instructions for the students:

- Attempt **all the questions** from section A.
- Solve **one question** of each unit from section B and any **three questions** from section C.
- For section A, each question carries **1.5 marks**. For section B, each question carries **8 marks**. For section C, each question carries **15 marks**.

Section-A (Objectives)

1. If X and Y are independent, then Covariance of X and Y is

(a) 1 (b) 0
(c) ∞ (d) none of these.

2. For any TPM, its eigen value is/are always

(a) 1 (b) 0 (c) both 0 and 1 (d) none of these

3. If F(x) be a distribution function of a random variable X, then F(x) is

(a) a bounded function (b) a monotonically non-decreasing function
(c) always a continuous function (d) all of these.

4. In Markov chain, a state-i is non-null-persistent, if

(a) $\mu_{ii} = \infty$ (b) $\mu_{ii} < \infty$ (c) $\mu_{ii} = 0$ (d) $\mu_{ii} = 1$.

5. The following distribution(s) function has *memory less property*

(a) Binomial distribution (b) Poisson distribution
(c) Exponential distribution (d) none of these.

6. If $X \sim Poisson(a)$ then, the Var(X) is given by

(a) $\frac{1}{a}$ (b) $\frac{1}{a^2}$ (c) $\frac{1}{a^3}$ (d) $\frac{1}{a^4}$

7. Poisson process is

(a) a stationary process (b) not a stationary process
(c) a covariance stationary process (d) none of these.

8. Probability generating function is

(a) not continuous (b) is not differentiable (c) infinitely differentiable (d) none of these.

9. Which is the correct answer?

(a) $L_S = \lambda W_S$ (b) $L_S = \mu W_S$ (c) $L_S = \lambda / W_S$ (d) $L_S = \mu / W_S$

10. In M/M/1 queueing model, P_0 is

(a) $(1-\rho)$. (b) $(1-\rho^2)$. (c) ρ. (d) $\frac{1}{1-\rho}$.

Section-B

Unit-1

1 Let X and Y be integer valued random variables with $P(X=m, Y=n) = q^2 p^{m+n-2},\ m, n = 1, 2, 3, \ldots$ and $p+q=1$. Are X and Y are independent?

2 Find the probability generating function of the random variable X whose probability function $P(X=x) = \frac{1}{3^x};\ x = 1, 2, 3, \ldots$ Also find it's variance.

Unit-2

1. Define the homogeneous Markov chain and its properties.
2. Write a TPM for any Markov chain.

Unit-3

1. Define the irreducibility and Ergodicity in Markov chain.
2. Define Regular Markov chain and give one example.

Unit-4

1. If X(t) denotes the number of events that takes place in an interval of length t, then under the axioms of Poisson process, show that the distribution of X(t) is Poisson with parameter λt.

2 Derive the formula for pure birth processes.

Unit-5

1. Find the expected length of non-empty queue for the multiple servers' queueing system with finite capacity.
2. Find the expected number of customers in the system for the M/M/K queueing system with infinite capacity.

Section-C

1. Derive the time dependent equations for birth-death process and calculate the mean population size also discuss the limiting cases, if $\lambda_n = n\lambda,\ \mu_n = n\mu\ (n \geq 1), \lambda_0 = 0,\ \mu_0 = 0$.
2. Derive the formula for steady-state probability distribution for M/G/1 queueing model. Find the Expected number of customers in the system by using probability distribution.
3. Two random variables X and Y have the joint probability density function $P(x,y) = \begin{cases} xe^{-x(y+1)}, & x \geq 0,\ \ y \geq 0 \\ 0, & otherwise \end{cases}$ Find,

   (i) The marginal probability density function of X and Y.

   (ii) The conditional distribution of Y for a given X and of X for a given Y.

4. Classify the Stochastic process and give the application with some examples.
5. Given the TPM with 3- states (1, 2, 3) of the Markov chain $\{X_n(t)\}$ as:

$$P = \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.5 & 0.2 & 0.3 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}$$ and the initial distribution is $P^{(0)} = [0.4,\ 0.5,\ 0.1]$.

Find (i) $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$