MA/ M.Sc. Applied Mathematics, IV- Semester, 2014

End - Semester examination

Course Title: Wavelet Analysis and Applications **Course no.:** MAMT 419

Time Allowed: 03 hrs **Maximum marks:** 100

Section(A)

Note: Attempt all questions. Each question carries one mark.

1. For $f \in L_2(\mathbb{R})$, which of the following is equal to $\widehat{\overline{f(t)}}(\omega)$?

a) $\overline{\hat{f}(-\omega)}$ b) $\hat{f}(-\omega)$

c) $\overline{\hat{f}(\omega)}$ d) $\hat{f}(\omega)$

2. If $F: L_2(\mathbb{R}) \to L_2(\mathbb{R})$ is Fourier transform, then F is

a) continuous b) linear

c) bijection d) all of the above

3. If $\{V_j: j \in \mathbb{Z}\}$ is MRA, then $\cap_{j\in\mathbb{Z}} V_j$ is equal to

a) $L_2(\mathbb{R})$ b) $\{0\}$

c) $\emptyset$ d) $\hat{f}(\omega)$

4. The scaling function for the Haar system is equal to

a) $1_{[0,1)}$ b) $1_{(0,1)}$

c) $1_{[0,1]}$ d) $1_{(0,1]}$

5. If $f: \mathbb{R} \to \mathbb{C}$ and $A = \{ t \in \mathbb{R}: f(t) \neq 0\}$, then support of f is equal to

a) A b) A^0

c) $\bar{A}$ d) none of the above

6. A frame is said to tight frame if the frame bounds A, B satisfy:

a) $A = B$ b) $A > B$

c) $A < B$ d) none of the above

7. If $g \in S(\mathbb{R})$ and $\hat{g}$ are windows, then $\Delta g\, \Delta \hat{g}$

a) $\Delta g\, \Delta \hat{g} \geq \frac{1}{2}$ b) $\Delta g\, \Delta \hat{g} \leq \frac{1}{2}$

c) $\Delta g\, \Delta \hat{g} = \frac{1}{2}$ d) none of the above

8. If φ is a scaling function having compact support and $\hat{\varphi}(0) \neq 0$, then

a) $m_\varphi(0) = 0$ b) $m_\varphi(0) = 1$

c) $m_\varphi(0) = -1$ d) none of the above

9. If $f \in L_1(\mathbb{R})$, then

a) $\hat{f} \in L_1(\mathbb{R})$ b) $\hat{f} \notin L_1(\mathbb{R})$

c) $\hat{f}$ *may or may not be in* $L_1(\mathbb{R})$ d) none of the above

10 In medicines,wavelets have been used for the analysis of

a) ECG b) MRI c) EEG d) all of the above

Section(B)

Note: Attempt any five questions. Each question carries six marks.

Q.no.01: Find the discrete Fourier transform of $g = (1, i, i^2, i^3)$.

Q.no.02: Explain Buneman's algorithm..

Q.no.03: Define the terms: (i) Filter (ii) Multiresolution analysis

Q.no.04: Show that a mother wavelet produces an orthonormal basis for $L_2(\mathbb{R})$.

Q.no.05: If φ is a scaling function with compact support and $\hat{\varphi}(0) \neq 0$, then show that its filter m_φ is a trigonometric polynomial which is continuous and 2π-periodic.

Q.no.06: Define the terms: (i) Short Fourier transform (ii) Continuous wavelet transform.

Q.no.07: Define discrete Fourier transform and show that it is a linear bijection.

Q.no.08: Write a note of applications of wavelets to medicine.

Section(C)

Note: Attempt any five questions, selecting one question from each unit. Each question carries twelve marks.

UNIT-01

Q.no.01: If $f \in L_2(\mathbb{R})$ is continuous and band limited ; $\hat{f}(\omega) = 0$ for $|\omega| > k$ for some constant k, then show that

$$f(t) = \sum_{n\in\mathbb{Z}} f\left(\frac{n\pi}{k}\right) \frac{sin\,(kt - n\pi)}{kt - n\pi}.$$

Q.no.02: Show that the functions $\{{}^{\omega_j}/_{\sqrt{N}} : j = 0,1, \dots, N-1\}$ form an orthonormal basis for $L_1(Z_N)$.

UNIT-02

Q.no.03: If $f \in L_2(\mathbb{R})$, then show that $\{f(t-n) : n \in \mathbb{Z}\}$ is orthonormal if and only if

$$\sum_{n\in\mathbb{Z}} \left|\hat{f}(\omega + 2n\pi)\right|^2 = 1 \;\; a.e.$$

Q.no.04: State and prove mother wavelet theorem.

UNIT-03

Q.no.05: If φ is a scaling function having compact support and $\hat{\varphi}(0) \neq 0$ with

$$m_\varphi(\omega) = \sum_{k=-n}^{n} \frac{c_k}{\sqrt{2}} e^{-ik\omega} = 1\,,$$

then show that

$$\prod_{j\in\mathbb{N}} m_\varphi\left(\frac{\omega}{2^j}\right)$$

Converges uniformly on bounded subsets of $\mathbb{R}$.

Q.no.06: Establish that the trigonometric polynomials are not sufficient to generate wavelets.

UNIT-04

Q.no.07: If S is the frame operator of a frame $\{x_n\}$, in a Hilbert space, with frame bounds A, B, then show that

(i) S^{-1} exists , is positive and satisfies $B^{-1}I \leq S^{-1} \leq A^{-1}I$

(ii) $\{S^{-1}(x_n)\}$ is a frame.

Q.no.08: Write a note on wavelet packets.

UNIT-05

Q.no.09: Discuss applications of wavelets to differential equations.

Q.no.10: Write a note on applications of wavelets to statistics.