M A/ M Sc Applied Mathematics, Central University of Jammu
Semester-II, End Semester Examination 2017

Course Title: Complex Analysis — **Course number**: PGAMT2C003T
Time Allowed: 3 hours — **Maximum Marks**: 100

Instructions for the candidates:

- The question paper consist of three sections, namely, **Section A**, **Section B** and **Section C**.
- The **section A** consist of 10 objective type questions, and all the questions are compulsory in this section.
- The **section B** consist of 10 short answer type questions with 2 questions from each unit, and the candidate has to attempt 5 questions selecting one question from each unit.
- The **section C** consist of 5 long answer type questions, and the candidate has to attempt any 3 questions.

Section A

(1) A necessary and sufficient condition that the two complex numbers Z_1 and Z_2 be parallel is

(a) $z_1 \times z_2 = 0$.
(b) $z_1 \cdot z_2 = 0$.
(c) Both (a) and (b).
(d) None of the above. 1.5

(2) Which of the following is true ?

(a) $|z_1 - z_2| \leq |z_1| - |z_2|$.
(b) $|z_1 - z_2| \geq |z_1| - |z_2|$.
(c) $|z_1 - z_2| = |z_1| - |z_2|$.
(d) None of the above. 1.5

(3) Let $f(z)$ be an analytic function in a simple connected region R. If a and z are two points in R and $F(z) = \int_a^z f(z)dz$, then

(a) $F(z)$ is analytic in R and $F'(z) = f(z)$.
(b) $F(z)$ may not be analytic in R and $F'(z) = f(z)$.
(c) $F(z)$ is analytic in R and $F'(z) \neq f(z)$.
(d) None of the above. 1.5

(4) Consider the following two statements:

(i) If $f(z)$ is analytic in a region R and on its boundary C. Then $\int_C f(z)dz = 0$
(ii) If $f(z)$ is continuous in a simply connected region R and $\oint_C f(z)dz = 0$ then $f(z)$ is analytic in R, then

(a) Both (i) and (ii) are correct.
(b) Only (i) is correct.
(c) Only (ii) is correct.
(d) None of the above. 1.5

(5) If $f(z)$ is analytic within and on the boundary C of a simply connected region R, then

(a) $f'(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$.
(b) $f'(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^2}dz$.

(c) $f^n(a) = \frac{n!}{2\pi} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$.

(d) None of the above. 1.5

(6) The winding number of $|z-1| = 1$ with respect to $z = 3$ is

(a) 0.

(b) 1.

(c) 2.

(d) None of the above. 1.5

(7) The residue of $f(z) = \frac{z^3}{z^2-1}$ at $z = 1$ is

(a) 1.

(b) $\frac{1}{2}$.

(c) $\frac{1}{3}$.

(d) $\frac{1}{4}$. 1.5

(8) The radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}(z-1-i)^n$ is

(a) 0.

(b) ∞.

(c) 1

(d) n. 1.5

(9) Consider the following two statements

(i) The function $f(z) = e^z$ is conformal at every point in $\mathbb{C}$.

(ii) $g(z) = z^2$ is conformal at $z = 0$, then

(a) Both (i) and (ii) are true.

(b) Only (i) is true.

(c) Only (ii) is true.

(d) None of the above. 1.5

(10) Consider the function $f(z) = z^3 - 3z + 1$, then

(a) $f(z)$ is conformal at $z = \pm 1$.

(b) $f(z)$ is not conformal at $z = \pm 1$.

(c) $f(z)$ is not conformal at $z = 0$.

(d) None of the above. 1.5

Section B

Unit - I

(1) Derive an expression for Cauchy-Riemann equations in polar form. 8

(2) Show that the real and imaginary parts of an analytic function $f(z) = u + iv$ satisfy Laplace equation. 8

Unit - II

(3) State and prove fundamental theorem of algebra. 8

(4) If $f(z)$ is analytic in a simply connected region R. Prove that $\int_a^b f(z)dz$ is independent of the path in R joining any two points a and b in R. 8

Unit - III

(5) State and prove Morera's theorem. 8

(6) State and prove Liouville's theorem. 8

Unit - IV

(7) Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 \cdot 4^n}$. 8

(8) State and prove Jordan's lemma. 8

Unit - V

(9) Show that the composition of two mobius transformations is a Mobius transformation. 8

(10) State and prove Schwartz's lemma. 8

Section - C

(11) Define multi-valued function and show that $f(z) = \sqrt{\frac{z}{z-1}}$ is a multi-valued function. 15

(12) State and prove Green's theorem for complex valued function. 15

(13) State and prove Cauchy's integral formula for higher order derivatives. 15

(14) Prove that

$$\int_0^\infty \frac{x^3}{x^4+a^4}\sin mx dx = \frac{\pi}{2}e^{\frac{-ma}{\sqrt{2}}}\cos\frac{ma}{\sqrt{2}}.$$

15

(15) Prove that the mapping $w = \frac{1}{z}$ transforms circle and straight lines into circle and straight lines. 15