MA/MSc Applied Mathematics, IInd-Semester, 2016-2017

End-Semester Examination

Course Title: Topology **Course Code:** PGAMT2C001T

Time allowed: 3hours **Maximum Marks:** 100

Instructions for the candidates:

- *The question paper consists of three sections, namely* **Section A, Section B** *and* **Section C**.
- *The* **section A** *consists of 10 objective type questions and all the questions are compulsory in this section.*
- *The* **section B** *consists of 10 short answer type questions with 2 questions from each unit and the candidate has to attempt 5 questions selecting one question from each unit.*
- *The* **section C** *consists of 5 long answer type questions one question from each unit and the candidate has to attempt any three questions.*

Note: (1) Topology on $\mathbb{R}^n, n \geq 1$ is usual topology unless a different one is specified.

(2) Meaning of a space is topological space unless mentioned.

Section A

1. Which of the following topological spaces is disconnected?
(a) Real line with usual topology.
(b) Set of non-zero real numbers with subspace topology of $\mathbb{R}$.
(c) Any trivial Space
(d) None of these 1.5

2. Given f: X → Y be a continuous function from topological space X onto topological space Y. Then
(a) If X is first countable, then Y is first countable.
(b) If X is separable, then Y is separable.
(c) If X is second countable, then Y is second countable.
(d) All of the above 1.5

3. Let X be a connected space. Then
(a) X is not the union of two separated sets.
(b)The only subsets of X which are both open and closed are $\phi$and X.
(c) X is not the union of two disjoint, non-empty closed sets.
(d) All of the above. 1.5

4. Which of the following subspace of $\mathbb{R}$ is locally connected but not connected?
(a) [1,5]

(b) Set of rationals.
(c) $[0,1] \cup [4,5]$
(d) None of these 1.5

5. The one point compactification $\mathbb{R}_\infty$ of the real line $\mathbb{R}$ is:
(a) A circle
(b) $S^2 = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : ||x|| = 1\}$
(c) n dimensional sphere
(d) None of these 1.5

6. Which of the following is not true for a topological space X?
(a) If X is compact then it is countably compact.
(b) If X is Lindelöf and compact then it is countably compact.
(c) If X is Lindelöf and countably compact then it is compact.
(d) If X is countably compact then it is compact. 1.5

7. The quotient space of $[0,1] \times [0,1]$ obtained by identifying the pair of points $(0, x_2)$ and $(1, 1 - x_2), 0 \leq x_2 \leq 1$ is homeomorphic to
(a) Torus
(b) Mobius Strip
(c) Circle
(d) Projective Plane 1.5

8. Which of the following is true?
(a) Product of any family of compact spaces is compact.
(b) Product of finite number of compact spaces is compact.
(c) Product of finite number of Hausdorff spaces is Hausdorff.
(d)All of these 1.5

9. A topological space X is said to be $T_{1\frac{1}{3}}$ space if
(a) Each sequence in X has at most one limit.
(b) Its each compact set is closed.
(c) Every finite subset of X is closed.
(d) All of above. 1.5

10. Which of the following is true for a topological space X?
(a) X is Hausdorff $\Rightarrow$ X is regular.
(b) X is T_1 $\Rightarrow$ X is normal.
(c) X is regular $\Rightarrow$ X is normal.
(d) None of these 1.5

Section B

Unit-I

1. Prove that every separable metric space is second countable. 8
2. Prove that a subset A of a topological space X is open if and only if $A = int\ A$. 8

Unit-II

1. Let X be a connected space and $f: X \to Y$ a continuous function from X onto a space Y. Prove that Y is connected. 8
2. State and prove The Intermediate Value Theorem. 8

Unit-III

1. Let X be a compact space, Y a Hausdorff space, and $f: X \to Y$ a continuous one-to-one function from X onto Y. Prove that f is a homeomorphism. 8
2. Prove that each closed subset of a compact space is compact. 8

Unit-IV

1. Prove that product of a finite number of connected spaces is connected. 8
2. Let X and Y be spaces and $f: X \to Y$ a continuous function from X onto Y. If f is either open or closed prove that Y has the quotient topology determined by f. 8

Unit-V

1. Prove that a T_1-space X is regular if and only if for each point a in X and each open set U containing a, there is an open set W containing a whose closure is contained in U. 8
2. Prove that every metric space is normal. 8

Section C

1. Prove that separability, first countability and second countability are topological properties. 15
2. Prove that $\mathbb{R}$ is connected and connected subsets of $\mathbb{R}$ are precisely the intervals. 15
3. Prove that a metric space is compact if and only if it has the Bolzano-Weierstrass property. 15
4. Prove that product of an arbitrary family of compact spaces is compact. 15
5. State and prove Tietze Extension Theorem. 15